BT-MRE-I&II(S)-05.18-1054 Reg. No. [ ]

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B.Tech. Degree I & II Semester Supplementary Examination in Marine Engineering May 2018

MRE 101 ENGINEERING MATHEMATICS I

(Prior to 2013 Scheme)

Time: 3 Hours Maximum Marks: 100

(5 × 20 = 100)

I. (a) Verify Legrange's mean value theorem for $f(x) = \sin x$ in $(0, \pi)$. (5)

(b) Evaluate $\lim_{x\to\pi/2} \sin x^{\tan x}$. (7)

(c) Show that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $4a\cos\theta/2$. (8)

OR

II. (a) A window has the form of a rectangle surmounted by a semi circle. If the perimeter is 40 ft. Find its dimensions so that the greatest amount of light may be admitted. (8)

(b) Find the asymptotes of $x^3 + y^3 = 3axy$. (6)

(c) If $y = \tan^{-1} x$ prove that $(1 + x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$. (6)

III. (a) If $z = f(x + ct) + \phi(x - ct)$ prove that $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$. (10)

(b) If $\sin u = \frac{x^2 y^2}{x + y}$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$. (10)

OR

IV. (a) If $\phi(cx - az, cy - bz) = 0$ show that $a\frac{\partial z}{\partial x} = b\frac{\partial z}{\partial y} = c$. (10)

(b) In polar coordinates $x = r\cos\theta$, $y = r\sin\theta$ show that $\frac{\partial(x, y)}{\partial(r, \theta)} = r$. (10)

V. (a) Find the vertex, focus and directrix of the parabola $y^2 - 2x - 6y + 5 = 0$. (10)

(b) Derive the standard equation of the parabola $y^2 = 4ax$. (10)

OR

VI. (a) Find the focus and length of latus rectum of the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$. (10)

(b) Find the equation of the asymptotes of the hyperbola $2x^2 - xy + 3y^2 - 9x + 16y - 8 = 0$. (10)

(P.T.O.)

VII. (a) Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\,dxdy$ and hence evaluate the same. (10)

(b) Evaluate $\iiint_V dxdydz$ where V is the volume of the tetra hedron whose vertices are (0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1). (10)

OR

VIII. (a) Using triple integration find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (10)

(b) Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{1 - y^2(1 + x^2)}}$. (10)

IX. (a) Show that the volume of the tetrahedron ABCD is $\frac{1}{6}\left[\overline{AB}, \overline{AC}, \overline{AD}\right]$. (10)

(b) Prove that $\left[BxC, CxA, AxB\right] = \left[ABC\right]^2$. (10)

OR

X. (a) If $r = \sqrt{x^2 + y^2 + z^2}$ Evaluate $\nabla^2(\log r)$. (10)

(b) Find the value of a, b, c so that the vector $\overline{F} = (x + y + az)i + (bx + 2y - z)j + (-x + cy + 2z)k$ may be irrotational. (10)

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